

# 1

# Functions and Models



# 1.4

## Graphing Calculators and Computers

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# Graphing Calculators and Computers

Graphing calculators and computers can give very accurate graphs of functions.

A graphing calculator or computer displays a rectangular portion of the graph of a function in a **display window** or **viewing screen**, which we refer to as a **viewing rectangle**.

The default screen often gives an incomplete or misleading picture, so it is important to choose the viewing rectangle with care.

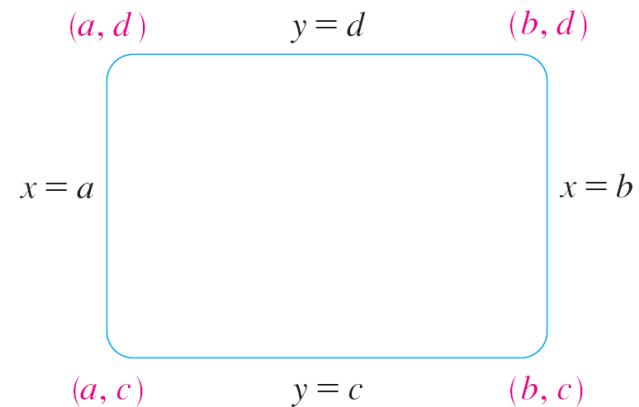
# Graphing Calculators and Computers

If we choose the  $x$ -values to range from a minimum value of  $Xmin = a$  to a maximum value of  $Xmax = b$  and the  $y$ -values to range from a minimum of  $Ymin = c$  to a maximum of  $Ymax = d$ , then the visible portion of the graph lies in the rectangle

$$[a, b] \times [c, d] = \{(x, y) \mid a \leq x \leq b, c \leq y \leq d\}$$

shown in Figure 1.

We refer to this rectangle as the  $[a, b]$  by  $[c, d]$  viewing rectangle.



The viewing rectangle  $[a, b]$  by  $[c, d]$   
Figure 1

# Graphing Calculators and Computers

The machine draws the graph of a function  $f$  much as you would.

It plots points of the form  $(x, f(x))$  for a certain number of equally spaced values of  $x$  between  $a$  and  $b$ .

If an  $x$ -value is not in the domain of  $f$ , or if  $f(x)$  lies outside the viewing rectangle, it moves on to the next  $x$ -value.

The machine connects each point to the preceding plotted point to form a representation of the graph of  $f$ .

# Example 1

Draw the graph of the function  $f(x) = x^2 + 3$  in each of the following viewing rectangles.

(a)  $[-2, 2]$  by  $[-2, 2]$

(b)(b)  $[-4, 4]$  by  $[-4, 4]$

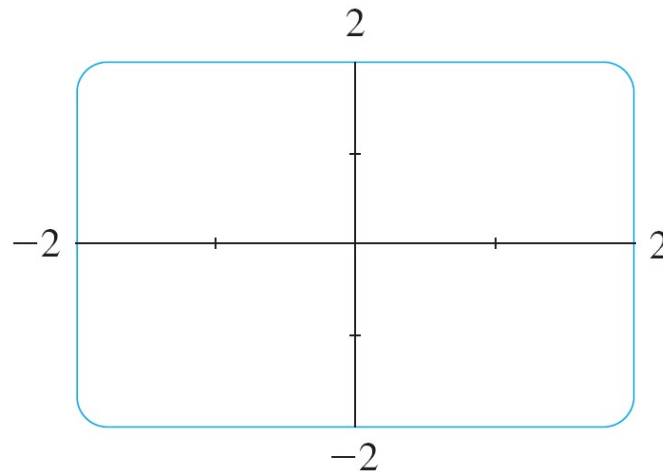
(c)  $[-10, 10]$  by  $[-5, 30]$

(d)  $[-50, 50]$  by  $[-100, 1000]$

# Example 1 – Solution

For part (a) we select the range by setting  $Xmin = -2$ ,  $Xmax = 2$ ,  $Ymin = -2$ , and  $Ymax = 2$ .

The resulting graph is shown in Figure 2(a). The display window is blank!



$[-2, 2]$  by  $[-2, 2]$   
Graph of  $f(x) = x^2 +$

3 **Figure 2(a)**

# Example 1 – *Solution*

cont'd

A moment's thought provides the explanation: Notice that  $x^2 \geq 0$  for all  $x$ , so  $x^2 + 3 \geq 3$  for all  $x$ .

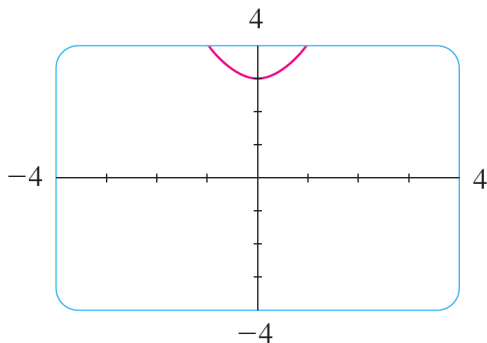
Thus the range of the function  $f(x) = x^2 + 3$  is  $[3, \infty)$ .

This means that the graph of  $f$  lies entirely outside the viewing rectangle  $[-2, 2]$  by  $[-2, 2]$ .

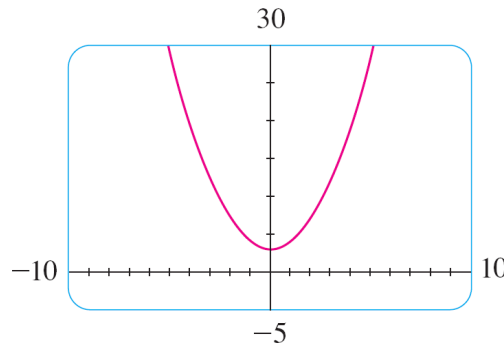
# Example 1 – Solution

cont'd

The graphs for the viewing rectangles in parts (b), (c), and (d) are also shown in Figure 2.



(b)  $[-4, 4]$  by  $[-4, 4]$

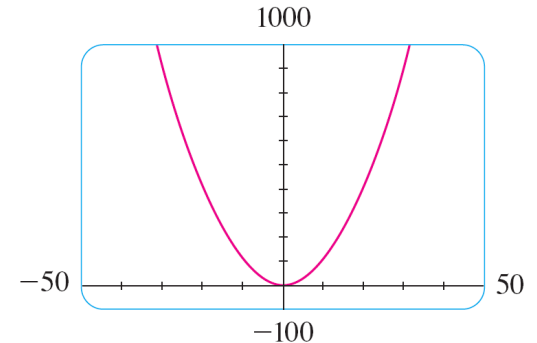


(c)  $[-10, 10]$  by  $[-5,$

$30]$   
Graphs of  $f(x) = x^2 +$

$3$

Figure 2



(d)  $[-50, 50]$  by  $[-100,$   
 $1000]$

Observe that we get a more complete picture in parts (c) and (d), but in part (d) it is not clear that the  $y$ -intercept is 3.

# Graphing Calculators and Computers

To understand how the expression for a function relates to its graph, it's helpful to graph a **family of functions**, that is, a collection of functions whose equations are related.

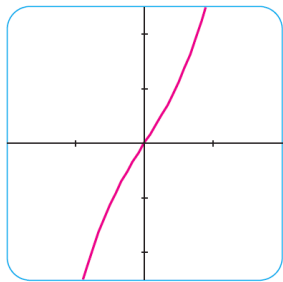
In the next example we graph members of a family of cubic polynomials.

# Example 8

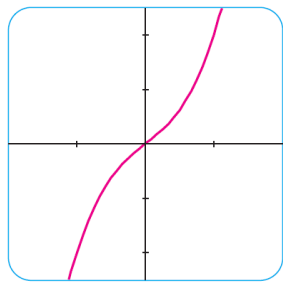
Graph the function  $y = x^3 + cx$  for various values of the number  $c$ . How does the graph change when  $c$  is changed?

**Solution:**

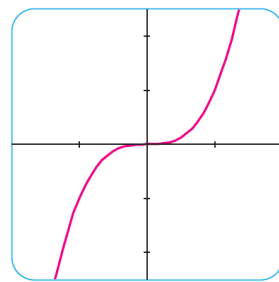
Figure 13 shows the graphs of  $y = x^3 + cx$  for  $c = 2, 1, 0, -1,$  and  $-2$ .



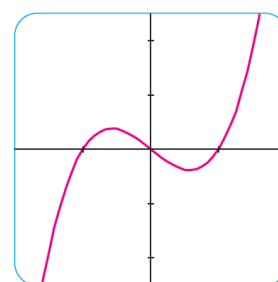
(a)  $y = x^3 + 2x$



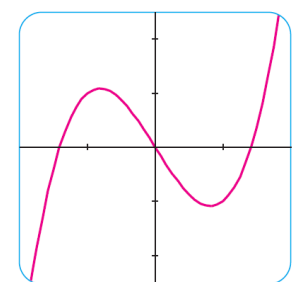
(b)  $y = x^3 + x$



(c)  $y = x^3$



(d)  $y = x^3 - x$



(e)  $y = x^3 - 2x$

Several members of the family of functions  $y = x^3 + cx$ , all graphed in the viewing rectangle  $[-2, 2]$  by  $[-2.5, 2.5]$

Figure 13

# Example 8 – *Solution*

cont'd

We see that, for positive values of  $c$ , the graph increases from left to right with no maximum or minimum points (peaks or valleys).

When  $c = 0$ , the curve is flat at the origin.

When  $c$  is negative, the curve has a maximum point and a minimum point.

As  $c$  decreases, the maximum point becomes higher and the minimum point lower.