7 Techniques of Integration

7.6 Integration Using Tables and Computer Algebra Systems

Tables of Integrals

Tables of Integrals

Tables of indefinite integrals are very useful when we are confronted by an integral that is difficult to evaluate by hand and we don’t have access to a computer algebra system.

Usually we need to use the Substitution Rule or algebraic manipulation to transform a given integral into one of the forms in the table.

Example 1

The region bounded by the curves *y* = arctan *x*, *y* = 0, and *x* = 1 is rotated about the *y*-axis. Find the volume of the resulting solid.

Solution:

Using the method of cylindrical shells, we see that the volume is

$$V = \int_0^1 2\pi x \arctan x \, dx$$

Example 1 – *Solution*

cont'd

In the section of the Table of Integrals titled *Inverse Trigonometric Forms* we locate Formula 92:

$$\int u \tan^{-1} u \, du = \frac{u^2 + 1}{2} \tan^{-1} u - \frac{u}{2} + C$$

Thus the volume is

$$V = 2\pi \int_0^1 x \tan^{-1} x \, dx = 2\pi \left[\frac{x^2 + 1}{2} \tan^{-1} x - \frac{x}{2} \right]_0^1$$

$$= \pi \left[(x^2 + 1) \tan^{-1} x - x \right]_0^1 = \pi (2 \tan^{-1} 1 - 1)$$

$$= \pi [2(\pi/4) - 1] = \tfrac{1}{2}\pi^2 - \pi$$

Computer Algebra Systems

Computer Algebra Systems

Computers are particularly good at matching patterns.

And just as we used substitutions in conjunction with tables, a CAS can perform substitutions that transform a given integral into one that occurs in its stored formulas.

So it isn’t surprising that computer algebra systems excel at integration.

To begin, let’s see what happens when we ask a machine to integrate the relatively simple function $y = 1/(3x - 2)$.

Computer Algebra Systems

Using the substitution $u = 3x - 2$, an easy calculation by hand gives

$$\int \frac{1}{3x - 2}\,dx = \tfrac{1}{3}\ln|3x - 2| + C$$

whereas Derive, Mathematica, and Maple all return the answer

$$\tfrac{1}{3}\ln(3x - 2)$$

The first thing to notice is that computer algebra systems omit the constant of integration.

Computer Algebra Systems

In other words, they produce a *particular* antiderivative, not the most general one.

Therefore, when making use of a machine integration, we might have to add a constant.

Second, the absolute value signs are omitted in the machine answer. That is fine if our problem is concerned only with values of *x* greater than $\frac{2}{3}$.

But if we are interested in other values of *x*, then we need to insert the absolute value symbol.

Example 5

Use a computer algebra system to find $\int x\sqrt{x^2 + 2x + 4}\, dx$.

Solution:

Maple responds with the answer

$$\tfrac{1}{3}(x^2 + 2x + 4)^{3/2} - \tfrac{1}{4}(2x + 2)\sqrt{x^2 + 2x + 4} - \frac{3}{2}\operatorname{arcsinh}\frac{\sqrt{3}}{3}(1 + x)$$

The third term can be rewritten using the identity

$$\operatorname{arcsinh} x = \ln(x + \sqrt{x^2 + 1})$$

Example 5 – *Solution*

cont'd

Thus

$$\operatorname{arcsinh}\frac{\sqrt{3}}{3}(1+x) = \ln\left[\frac{\sqrt{3}}{3}(1+x) + \sqrt{\tfrac{1}{3}(1+x)^2 + 1}\right]$$

$$= \ln\frac{1}{\sqrt{3}}\left[1 + x + \sqrt{(1+x)^2 + 3}\right]$$

$$= \ln\frac{1}{\sqrt{3}} + \ln\left(x + 1 + \sqrt{x^2 + 2x + 4}\right)$$

The resulting extra term $-\frac{3}{2}$ ln(1/ $\sqrt{3}$) can be absorbed into the constant of integration.

Example 5 – *Solution*

cont'd

Mathematica gives the answer

$$\left(\frac{5}{6} + \frac{x}{6} + \frac{x^2}{3}\right)\sqrt{x^2 + 2x + 4} - \frac{3}{2}\operatorname{arcsinh}\left(\frac{1 + x}{\sqrt{3}}\right)$$

Mathematica combined the first two terms of the Maple result into a single term by factoring.

Derive gives the answer

$$\tfrac{1}{6}\sqrt{x^2 + 2x + 4}\,(2x^2 + x + 5) - \tfrac{3}{2}\ln\left(\sqrt{x^2 + 2x + 4} + x + 1\right)$$